day 76

Math League tomorrow

Practice AP begins tomorrow

Jan 6-7:28 AM

"Finding the Area Between 2 Curves" day 76

Recall

$$\text{Area}^{*} = \int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} \Delta x_i \, f(x_i^{*})$$

* Area of a region
bounded on the left by $x=a$
bounded above by positive-valued f^n $y=f(x)$
bounded on the right by $x=b$
bounded below by x-axis

Jan 6-7:28 AM

day 76

$$\text{Net Area} = \int_a^b f(x)\,dx = \lim_{\substack{\max \Delta x_i \\ \to 0}} \sum_{i=1}^{n} \Delta x_i \, f(x_i^*)$$

1 factor goes to zero in the limit

other factor doesn't need to go to zero
bounded

generalization:
f(x) can be positive-valued, negative-valued or zero

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1) Area Between Two Curves

day 76

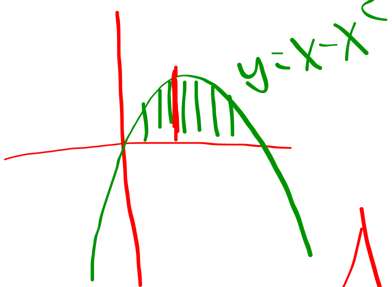
Area between $y=x$, $y=x^2$

$x = x^2$

$0 = x^2 - x$

$0 = x(x-1)$

$x = 0, +1$



Cavalieri's principle

$$\text{Area} = \int_0^1 x - x^2\,dx$$

$$= \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1$$

$$= \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{0}{2} - \frac{0}{3}\right) = \boxed{\frac{1}{6}}$$

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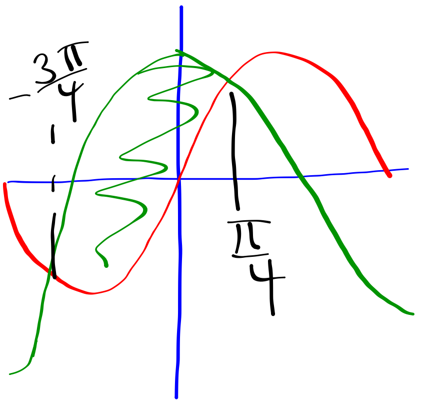
day 76

Find the area between

$y = \sin x$ and $y = \cos x$

in the single region closest to the y-axis.



$y = \cos x - \sin x$ on $\left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

$$\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} (\cos x - \sin x)\,dx = (\sin x + \cos x)\Big|_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} =$$

$$\left(\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)\right) - \left(\sin\left(-\frac{3\pi}{4}\right) + \cos\left(-\frac{3\pi}{4}\right)\right) =$$

$$\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} - \left(-\frac{\cancel{2}\sqrt{2}}{\cancel{2}}\right)$$

$$\sqrt{2} - (-\sqrt{2}) = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

day 76
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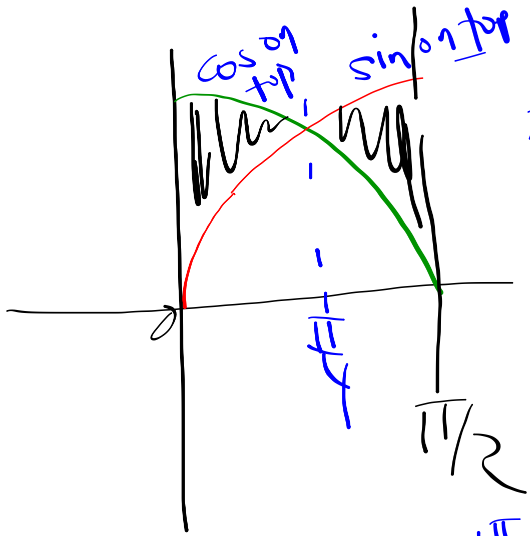
Find the area between

$y = \sin x$ and $y = \cos x$

between $x = 0$ and $x = \frac{\pi}{2}$.



$$\text{Area} = \int_0^{\pi/4} \cos x - \sin x \, dx + \int_{\pi/4}^{\pi/2} \sin x - \cos x \, dx$$

$$= (\sin x + \cos x)\Big|_0^{\pi/4} + (-\cos x - \sin x)\Big|_{\pi/4}^{\pi/2}$$

$$= \left(\sin\tfrac{\pi}{4} + \cos\tfrac{\pi}{4}\right) - (\sin 0 + \cos 0) + \left(-\cos\tfrac{\pi}{2} - \sin\tfrac{\pi}{2}\right) - \left(-\cos\left(\tfrac{\pi}{4}\right) - \sin\left(\tfrac{\pi}{4}\right)\right)$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (1) + (-0 - 1) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$\sqrt{2} - 1 - 1 + \sqrt{2} = 2\sqrt{2} - 2$$

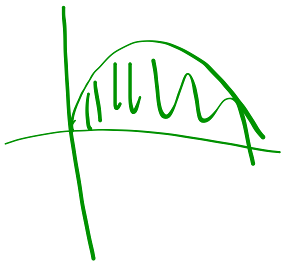
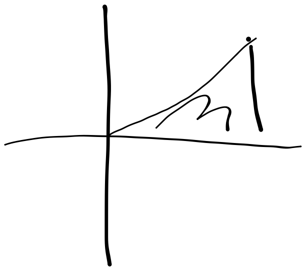
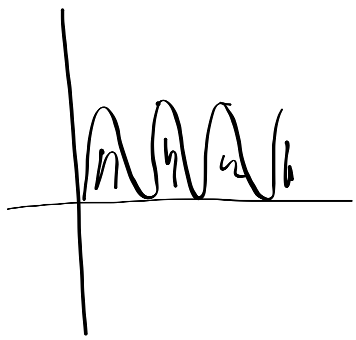
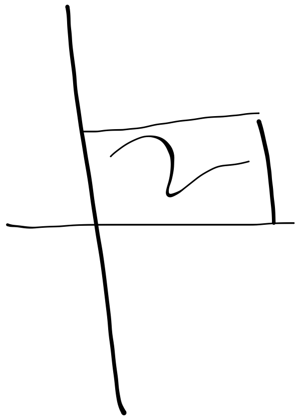
$$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$$

day 76

$$\text{Area} = \int_a^b f(x)\,dx = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} \Delta x_k\, f(x_k^*)$$

Notice → the Riemann Sum is an infinite-like sum of the product of two factors: one approaching 0 & one not

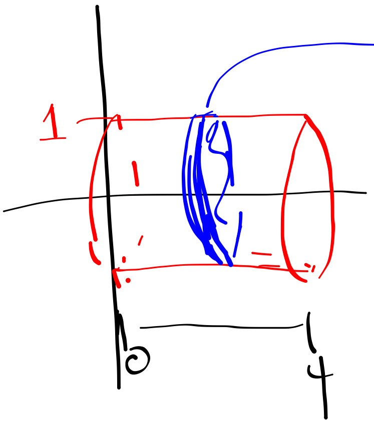
Notice → the definite integral is an infinite-like sum of heights of a function

Jan 6-7:28 AM

day 76

$$\underline{\text{Volume}} = \int_a^b \boxed{\text{Area of cross section}}\ \boxed{dx}$$



V of a slice $= (\pi r^2)\cdot(\Delta x)$

what the heck is r?

$r = (y = 1)$

Area of cross section

$$= \pi r^2 = \pi y^2 = \pi(1)^2 = \pi$$

$$V = \int_0^4 \pi\, dx = \pi x \Big|_0^4 = 4\pi - 0\pi = 4\pi$$